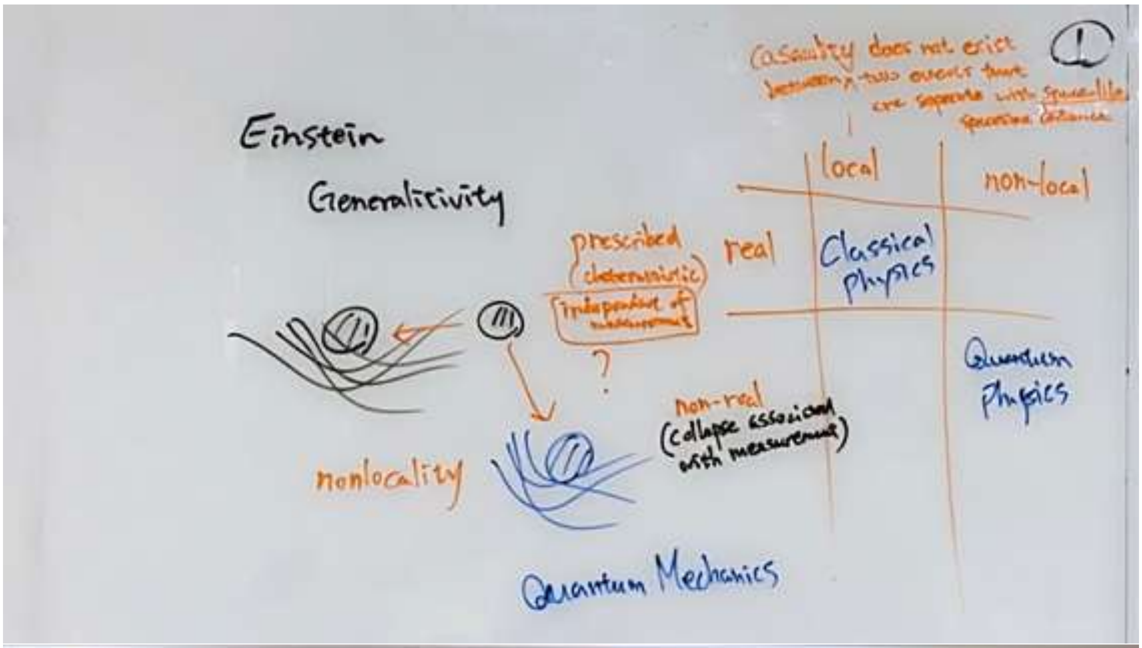
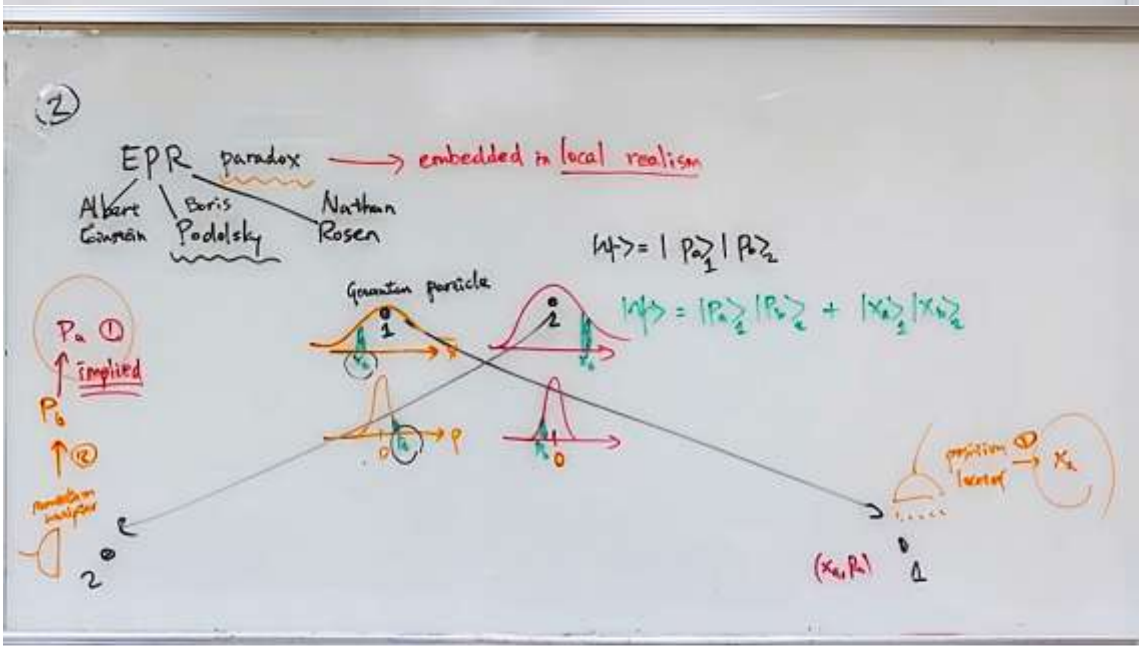
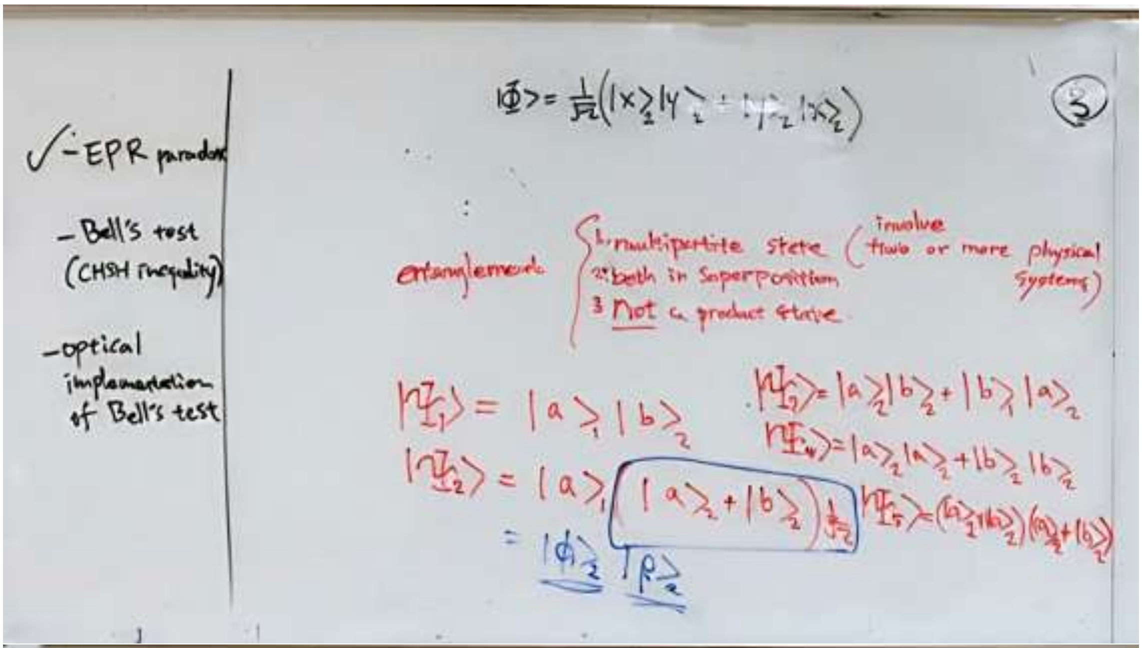
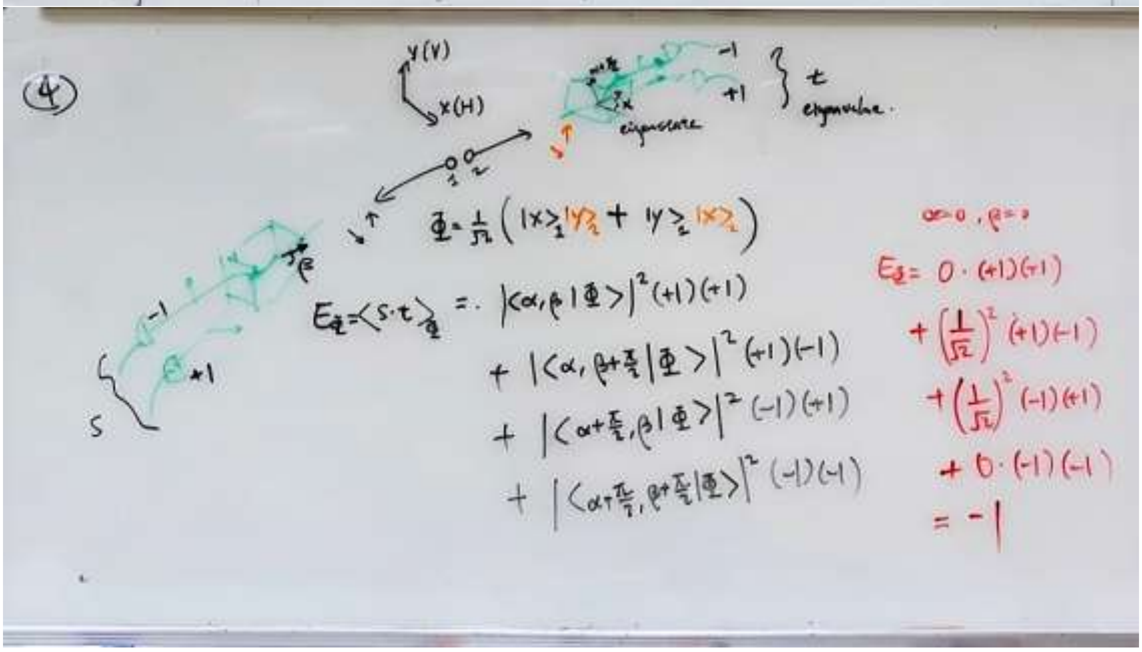
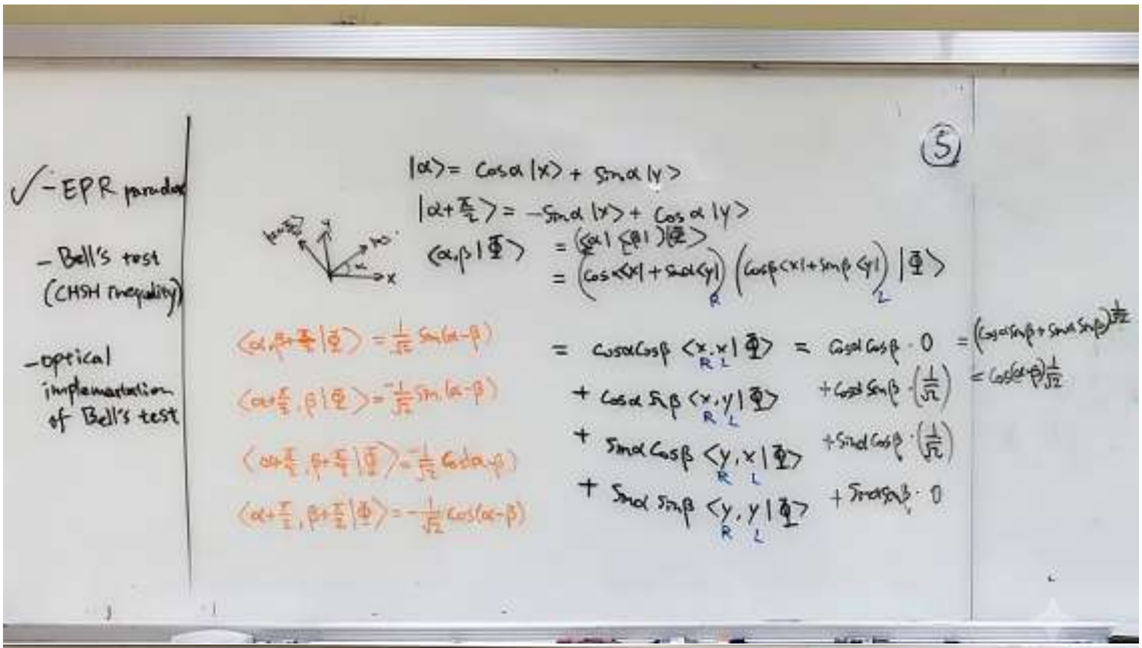
Lecture15 (optics) Bell's test and polarization entanglement

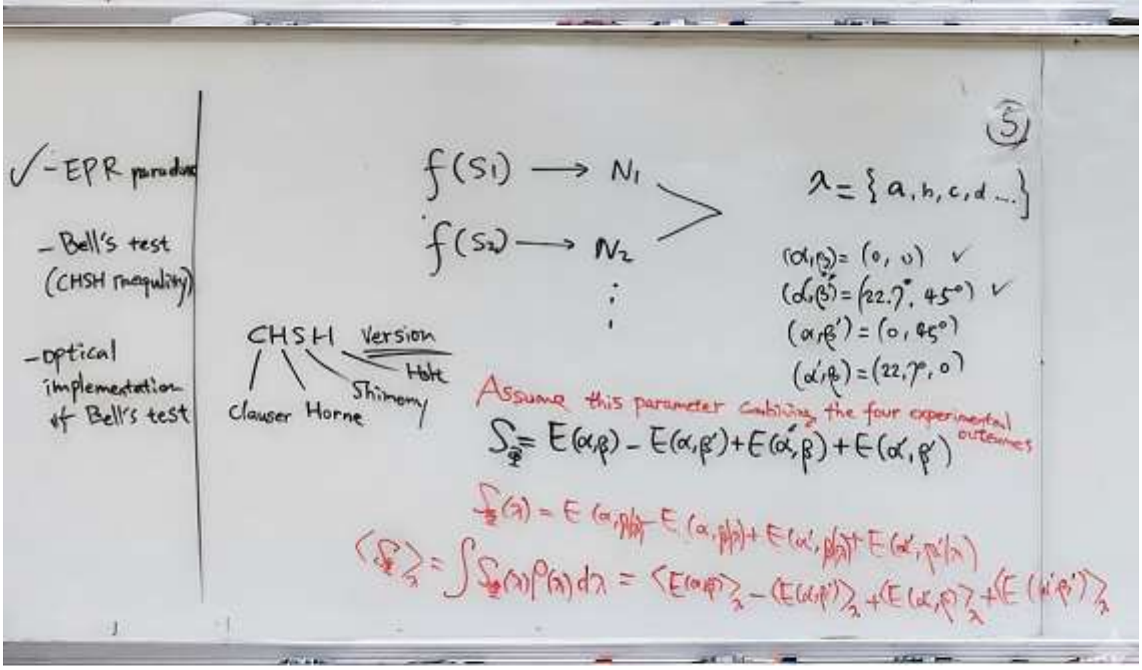


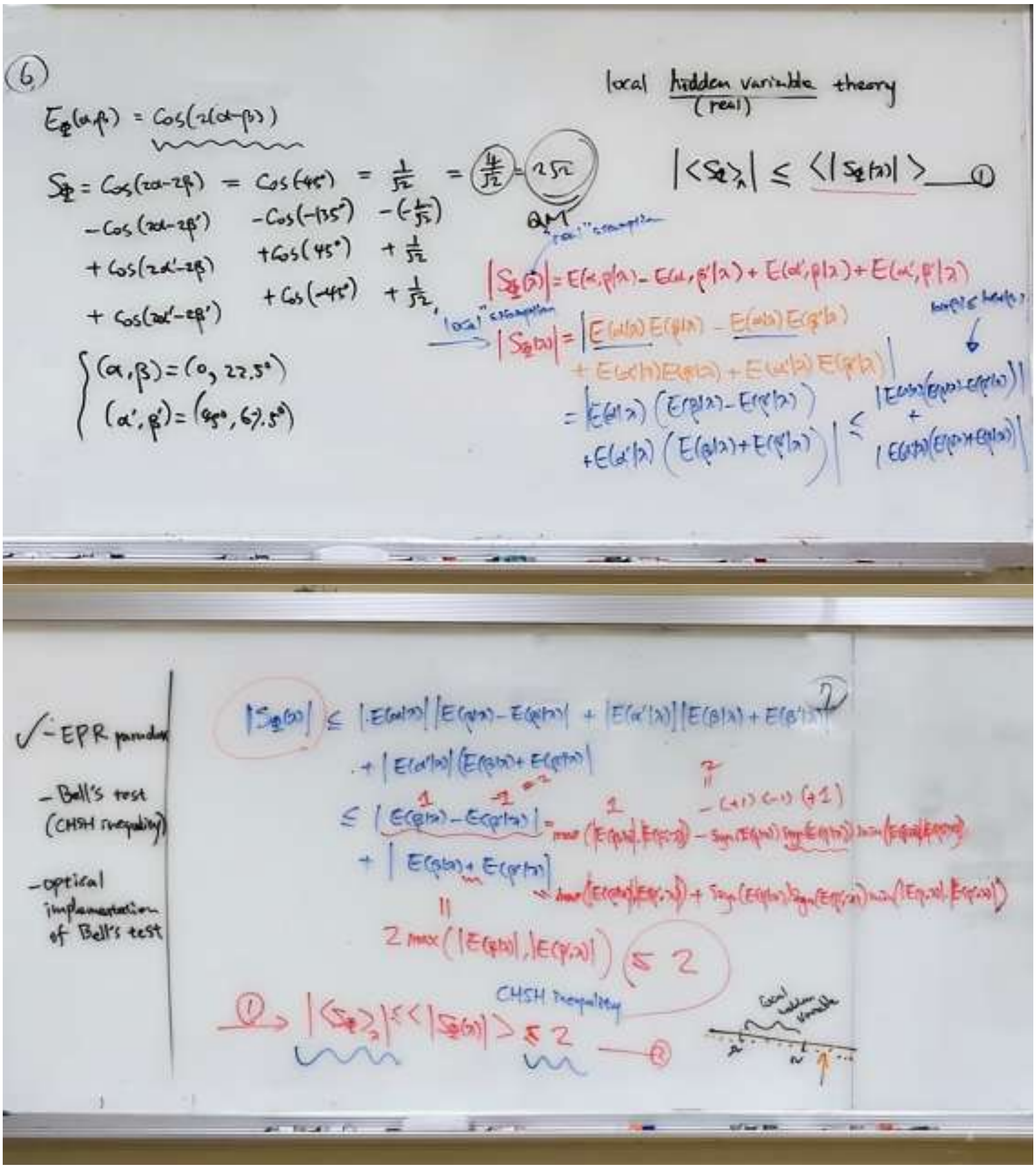


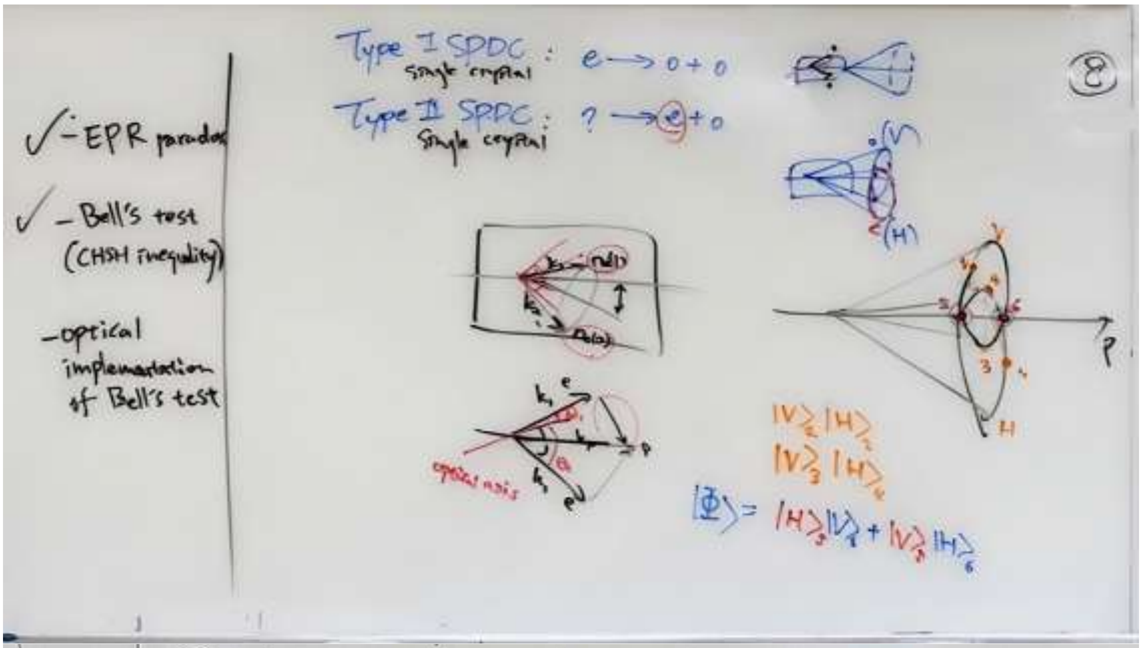

3
√ - EPR paradox
- Bell's test (CHSH inequality)
- optical implementation of Bell's test
$|\Phi\rangle = \frac{1}{\sqrt{2}}(|x\rangle_1|y\rangle_2 + |y\rangle_1|x\rangle_2)$
entanglement
1. multipartite state (involve two or more physical systems)
2. both in superposition
3 not a product state
$|\Psi_1\rangle = |a\rangle_1|b\rangle_2$
$|\Psi_2\rangle = |a\rangle_1 \frac{1}{\sqrt{2}}(|a\rangle_2 + |b\rangle_2)$
$= |\phi\rangle_1 |\rho\rangle_2$
$|\Psi_3\rangle = |a\rangle_1|b\rangle_2 + |b\rangle_1|a\rangle_2$
$|\Psi_4\rangle = |a\rangle_1|a\rangle_2 + |b\rangle_1|b\rangle_2$
$|\Psi_5\rangle = (|a\rangle_1 + |b\rangle_1)(|a\rangle_2 + |b\rangle_2)$

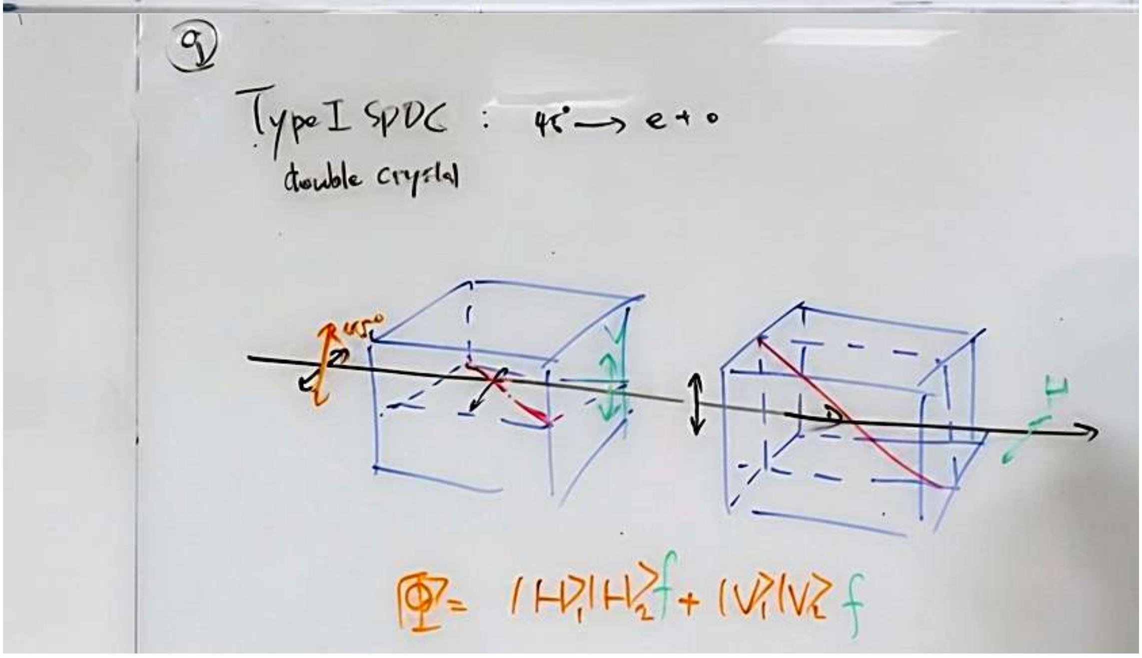

4
y(V)
x(H)
-1
+1
t eigenvalue
eigenstate
1 2
$\Phi = \frac{1}{\sqrt{2}}(|x\rangle_1|y\rangle_2 + |y\rangle_1|x\rangle_2)$
-1
+1
s
$E_\Phi = \langle s\cdot t\rangle_\Phi = |\langle\alpha,\beta|\Phi\rangle|^2(+1)(+1)$
$+ |\langle\alpha,\beta+\frac{\pi}{2}|\Phi\rangle|^2(+1)(-1)$
$+ |\langle\alpha+\frac{\pi}{2},\beta|\Phi\rangle|^2(-1)(+1)$
$+ |\langle\alpha+\frac{\pi}{2},\beta+\frac{\pi}{2}|\Phi\rangle|^2(-1)(-1)$
$\alpha=0, \beta=0$
$E_\Phi = 0\cdot(+1)(+1)$
$+ \left(\frac{1}{\sqrt{2}}\right)^2(+1)(-1)$
$+ \left(\frac{1}{\sqrt{2}}\right)^2(-1)(+1)$
$+ 0\cdot(-1)(-1)$
$= -1$

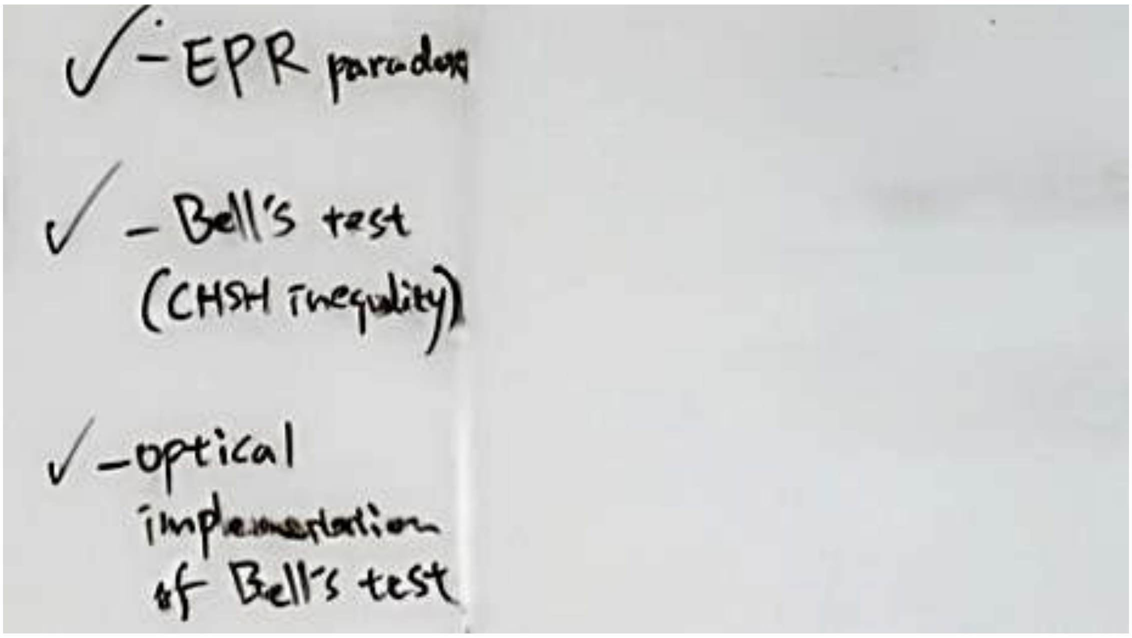

5
✓ - EPR paradox
- Bell's test (CHSH inequality)
- optical implementation of Bell's test
|α> = Cosα |x> + Sinα |y>
|α+π/2> = −Sinα |x> + Cosα |y>
<α,β|Φ> = (<α| <β|) |Φ>
= (Cosα<x| + Sinα<y|)R (Cosβ<x| + Sinβ<y|)L |Φ>
= CosαCosβ <x,x|Φ> = CosαCosβ · 0
+ CosαSinβ <x,y|Φ> + CosαSinβ · (1/√2)
+ SinαCosβ <y,x|Φ> + SinαCosβ · (1/√2)
+ SinαSinβ <y,y|Φ> + SinαSinβ · 0
= (CosαSinβ + SinαSinβ) 1/√2
= Cos(α−β) 1/√2
<α, β+π/2|Φ> = 1/√2 Sin(α−β)
<α+π/2, β|Φ> = −1/√2 Sin(α−β)
<α+π/2, β+π/2|Φ> = −1/√2 Cos(α−β)
<α+π/2, β+π/2|Φ> = −1/√2 Cos(α−β)


5
✓ - EPR paradox
- Bell's test (CHSH inequality)
- optical implementation of Bell's test
f(S1) → N1
f(S2) → N2
λ = {a, b, c, d ...}
(α,β) = (0, 0) ✓
(α',β') = (22.7°, 45°) ✓
(α,β') = (0, 45°)
(α',β) = (22.7°, 0)
CHSH Version
Clauser Horne Shimony Holt
Assume this parameter combining the four experimental outcomes
S_Φ = E(α,β) − E(α,β') + E(α',β) + E(α',β')
<S_Φ>_λ = ∫ S_Φ(λ) ρ(λ) dλ = <E(α,β)>_λ − <E(α,β')>_λ + <E(α',β)>_λ + <E(α',β')>_λ


(6)
$E_\Phi(\alpha,\beta) = \cos(2(\alpha-\beta))$
$S_\Phi = \cos(2\alpha-2\beta) - \cos(2\alpha-2\beta') + \cos(2\alpha'-2\beta) + \cos(2\alpha'-2\beta')$
$= \cos(45^\circ) - \cos(-135^\circ) + \cos(45^\circ) + \cos(-45^\circ)$
$= \frac{1}{\sqrt{2}} - (-\frac{1}{\sqrt{2}}) + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$
QM
$(\alpha,\beta) = (0, 22.5^\circ)$
$(\alpha',\beta') = (45^\circ, 67.5^\circ)$
local hidden variable theory (real)
$|\langle S_\Phi\rangle_\lambda| \le \langle |S_\Phi(\lambda)| \rangle$ — ①
"real" assumption
$|S_\Phi(\lambda)| = E(\alpha,\beta|\lambda) - E(\alpha,\beta'|\lambda) + E(\alpha',\beta|\lambda) + E(\alpha',\beta'|\lambda)$
"local" assumption
$|S_\Phi(\lambda)| = |E(\alpha|\lambda)E(\beta|\lambda) - E(\alpha|\lambda)E(\beta'|\lambda) + E(\alpha'|\lambda)E(\beta|\lambda) + E(\alpha'|\lambda)E(\beta'|\lambda)|$
$= |E(\alpha|\lambda)(E(\beta|\lambda)-E(\beta'|\lambda)) + E(\alpha'|\lambda)(E(\beta|\lambda)+E(\beta'|\lambda))|$
$|a+b| \le |a|+|b|$
$\le |E(\alpha|\lambda)(E(\beta|\lambda)-E(\beta'|\lambda))| + |E(\alpha'|\lambda)(E(\beta|\lambda)+E(\beta'|\lambda))|$
✓ − EPR paradox
− Bell's test (CHSH inequality)
− optical implementation of Bell's test
$|S_\Phi(\lambda)| \le |E(\alpha|\lambda)||E(\beta|\lambda)-E(\beta'|\lambda)| + |E(\alpha'|\lambda)||E(\beta|\lambda)+E(\beta'|\lambda)|$
$\le |E(\beta|\lambda)-E(\beta'|\lambda)| = \max(|E(\beta|\lambda)|,|E(\beta'|\lambda)|) - \mathrm{sign}(E(\beta|\lambda))\,\mathrm{sign}(E(\beta'|\lambda))\min(|E(\beta|\lambda)|,|E(\beta'|\lambda)|)$
$+ |E(\beta|\lambda)+E(\beta'|\lambda)| = \max(|E(\beta|\lambda)|,|E(\beta'|\lambda)|) + \mathrm{sign}(E(\beta|\lambda))\,\mathrm{sign}(E(\beta'|\lambda))\min(|E(\beta|\lambda)|,|E(\beta'|\lambda)|)$
$= 2\max(|E(\beta|\lambda)|,|E(\beta'|\lambda)|) \le 2$
CHSH inequality
① → $|\langle S_\Phi\rangle_\lambda| \le \langle |S_\Phi(\lambda)| \rangle \le 2$ — ②
local hidden variable
2
$2\sqrt{2}$


Type I SPDC : e → o + o
single crystal
Type II SPDC : ? → e + o
single crystal
⑧
✓ - EPR paradox
✓ - Bell's test
(CHSH inequality)
- optical implementation of Bell's test
optical axis
|V⟩₂ |H⟩₂
|V⟩₃ |H⟩₄
|Φ⟩ = |H⟩₃|V⟩₄ + |V⟩₃|H⟩₆


⑨
Type I SPDC : 45° → e + o
double crystal
45°
|Φ⟩ = |H⟩₁|H⟩₂ f + |V⟩₁|V⟩₂ f


✓ - EPR paradox
✓ - Bell's test
(CHSH inequality)
✓ - optical
implementation
of Bell's test